cc-minors in graphs and matroids

Wayne Ge
joint with James Oxley

LSU Combinatorics Seminar, August 2025

Operations and relations

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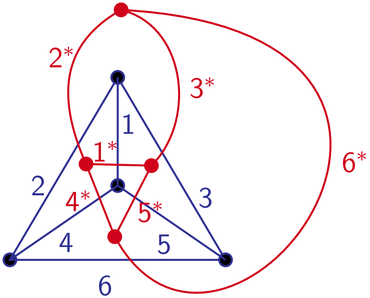

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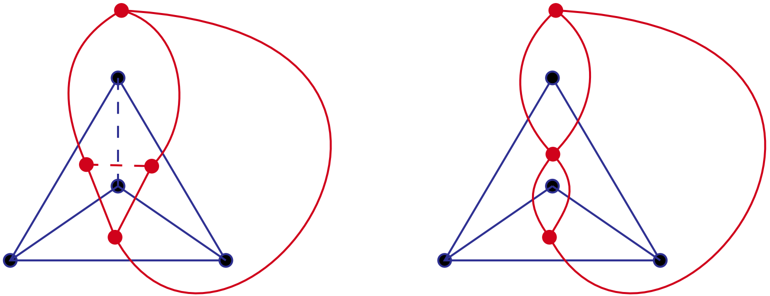

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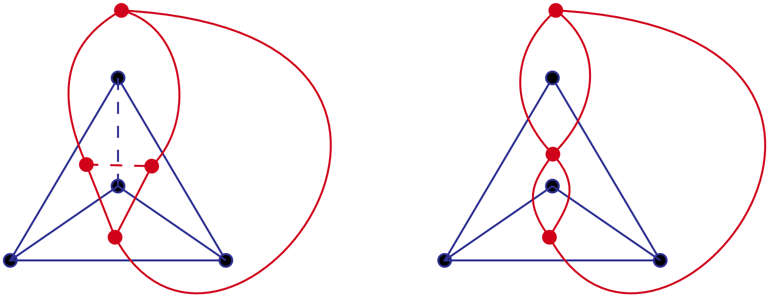

Proposition

Let G be a plane graph, $G \setminus e = (G^*/e^*)^*$.

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Does vertex deletion have a dual operation?

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Proposition

N is an induced restriction of M if and only if N^* is a cc-minor of M^*.

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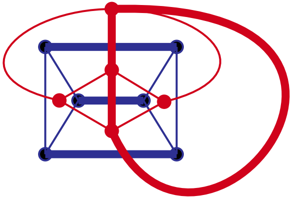
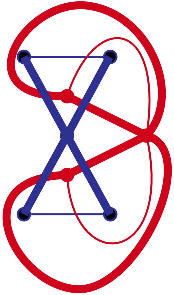
However, deleting a vertex is not in general dual to contracting a cycle.





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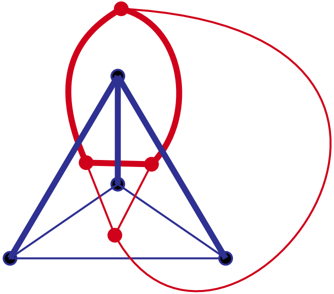

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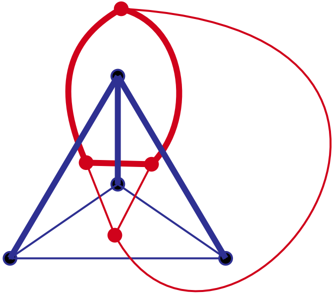


- $G - v = (G^*/E_v^*)^*$.

Duality

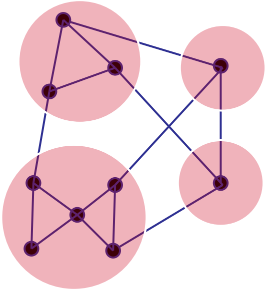
Lemma

Let G be a loopless 2-connected plane graph. A graph H is a 2-connected induced subgraph of G if and only if H^* is a 2-connected cc-minor of G^*.

From a minor perspective

Lemma

A graph G has a graph H as a **minor** *if and only if G has a set $\{G_v : v \in V(H)\}$ of disjoint* **connected** *subgraphs and a set $\{f_e : e \in E(H)\}$ of distinct edges that is disjoint from $\cup_{v \in V(H)} E(G_v)$ such that, for every edge $e \in E(H)$ having ends u and v, the ends of f_e are contained in G_u and G_v, respectively.*

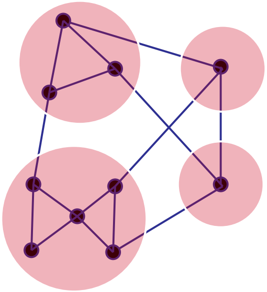


Lemma

A graph G contains a graph H as a **cc-minor** if and only if G has

(i) a collection $\{G_v : v \in V(H)\}$ of disjoint subgraphs such that each G_v is either a 2-**edge-connected** subgraph of G or a **single-vertex** subgraph of G, and $\bigcup_{v \in V(H)} V(G_v) = V(G)$; and

(ii) a set $\{f_e : e \in E(H)\} = E(G) - \bigcup_{v \in V(H)} E(G_v)$ such that, for every edge $e \in E(H)$ with endpoints u and v, the endpoints of f_e lie in G_u and G_v, respectively.



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What graph classes are closed under taking cc-minors?

Let $\mathcal{F}_k$ be the class consisting of all k-**edge-connected** graphs along with the single-vertex graph (possibly with loops).

Proposition

For every positive integer k, the class $\mathcal{F}_k$ is closed under cycle contractions.

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(iv) For $k \geq 4$, G is in $\mathcal{F}_k$ if and only if G is in $\mathcal{F}_{k-1}$ and G does not have a cc-minor isomorphic to B_{k-1}.

Ramsey properties and connectivity

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Let $\mathcal{F}$ be a family of graphs. An *unavoidable-families characterization* consists of a **subfamily** $\mathcal{U}$ of $\mathcal{F}$ and a **relation** $\leq$ such that, for every sufficiently large graph G in $\mathcal{F}$, there is a large graph H in $\mathcal{U}$ such that $H \leq G$.

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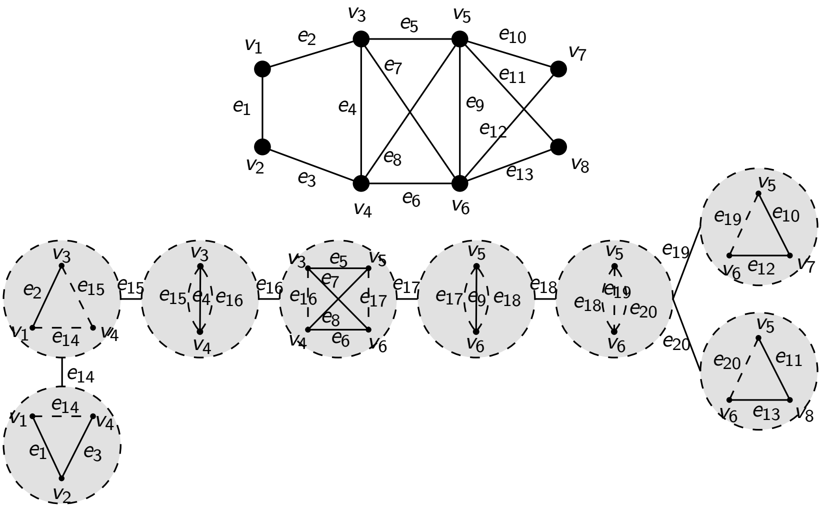
Theorem (Allred, Ding, Oporowski 2023)

Every giant simple 2-**connected** graph contains a large induced ...

Tree decomposition of 2-connected graph



Tree decomposition of 2-connected graph

Theorem (Tutte 1966)

Every 2-connected graph G has a tree decomposition in which every vertex label is a simple 3-connected graph, a copy of K_3, or a copy of B_3.

r-template

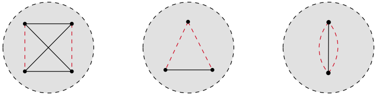
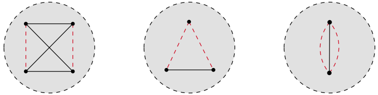


Figure: Building blocks of templates

r-template



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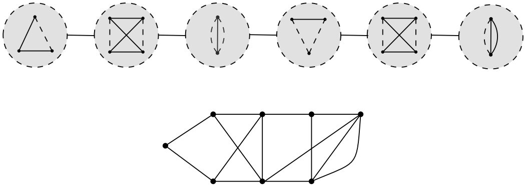

Figure: A sample 6-template

r-template

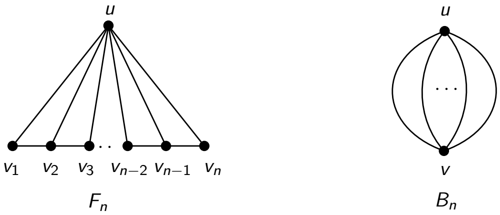

Figure: A fan graph F_n and a bond graph B_n.

Examples

(i) A **fan graph** F_n is a $(2n-3)$-template in which the parts alternate between K_3 and B_3, beginning and ending with K_3.

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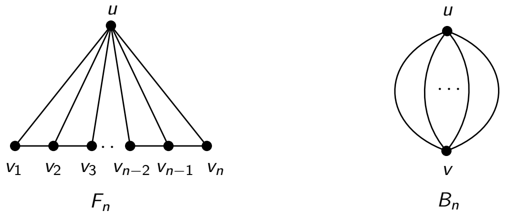

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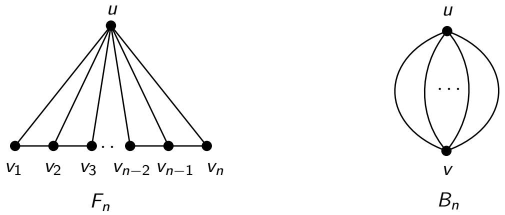

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(iii) For $n \geq 3$, a **cycle** C_n is an $(n-2)$-template for which every part is a K_3.

Ramsey properties and connectivity

Theorem (Allred, Ding, Oporowski 2023)

Every giant simple 2-**connected** graph contains a large induced K_r or an induced subdivision of an r-template.

Main result (a dual theorem)

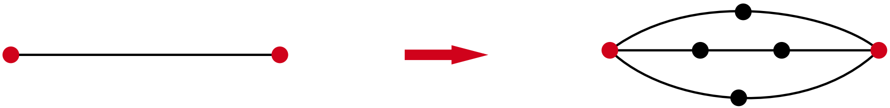

Figure: parallel-path extension

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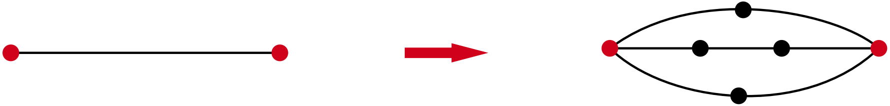

Figure: parallel-path extension

Theorem

Every giant 2-connected graph contains a **parallel-path extension** of a large r-template as a cc-minor.

Proving the main result

Case I

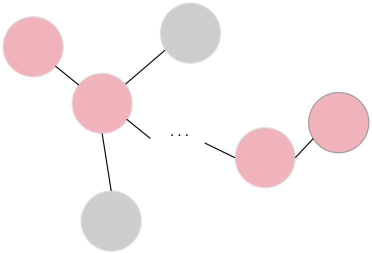

Figure: The tree decomposition has a long path

Case II

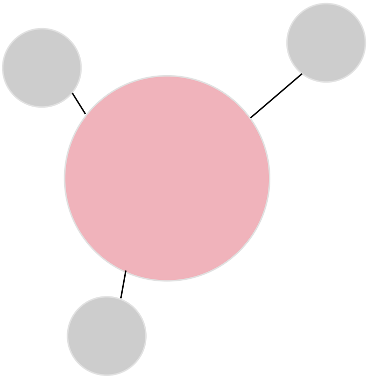

Figure: The tree decomposition has a giant vertex

Proving the main result

Lemma

Let G be a 2-edge-connected graph, and let e be an edge of G. Then G has a cc-minor H that is the **parallel connection**, with basepoint e, of a collection of cycles containing e.

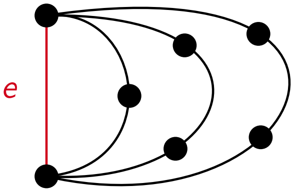

Figure: parallel connection of three cycles with basepoint e

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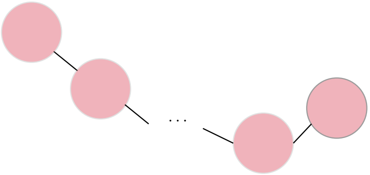

Case II

Proving the main result

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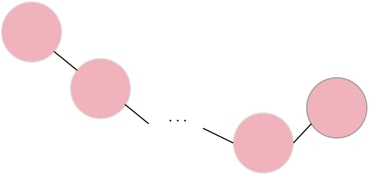

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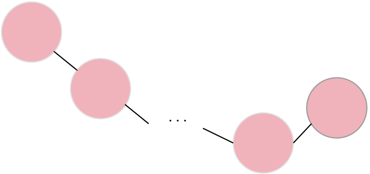
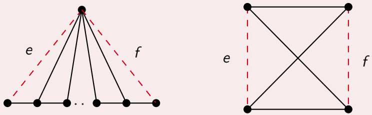


Lemma

Let G be a simple 3-connected graph, and let e and f be two distinct edges. Then G has a cc-minor H containing e and f such that the simplification of H is one of the following.



Proving the main result

Case II

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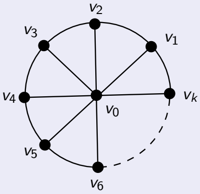
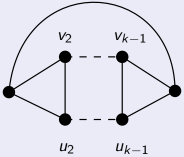
Theorem (Oporowski, Oxley, Thomas 1993)

A giant 3-connected graph contains a large subgraph isomorphic to a subdivision of one of W_k, V_k, and $K_{3,k}$.



k-spoke wheel W_k



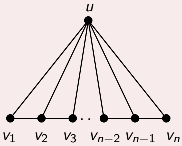
V_k

Proving the main result

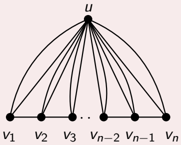
Case II

Lemma

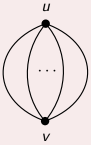
A giant 3-connected graph G contains a large fan-type graph as cc-minor.



$F_{1,1,\ldots,1}$



$F_{2,2,\ldots,2}$



B_n

Ramsey theory in matroids

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A (2-)connected matroid with more than $\frac{1}{2}cc^*$ elements has either a circuit of size greater than c or a cocircuit of size greater than c^*.

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A simple GF(q)-representable matroid M with sufficiently high rank has a rank-k flat which is either independent in M, or is a projective or affine geometry.

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A simple GF(q)-representable matroid M with sufficiently high rank has a rank-k flat which is either independent in M, or is a projective or affine geometry.

Find a set of unavoidable induced restrictions for simple 2-connected GF(q)-representable matroid.

From regular matroids to graphs

Find a set of unavoidable induced restrictions for sufficiently large simple 2-connected regular matroids.

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A matroid is **regular** if and only if it can be constructed from **graphic** matroids, **cographic** matroids, and **copies of** R_{10} by using sequences of the operations of direct sum, 2-sum, and 3-sum.

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In particular, a 2-connected cographic matroid $M(G)^*$ has a 2-connected induced restriction $M(H)^*$ if and only if G has H as a cc-minor.

Relaxed cc-minor

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Relaxed circuit-contraction minor

N is an relaxed cc-minor of M if it can be obtained by a sequence of circuit contractions and deletions.

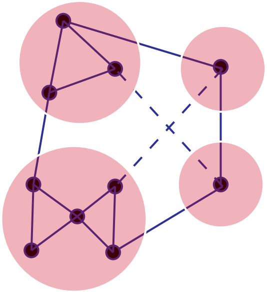
Relaxed cycle-contraction minor

H is a relaxed c-minor of G if it can be obtained by a sequence of cycle contractions and edge deletions.

Relaxed cc-minor

Lemma

A graph G has a graph H as a **relaxed cc-minor** *if and only if G has a set $\{G_v : v \in V(H)\}$ of disjoint* **2-edge-connected or single-vertex** *subgraphs and a set $\{f_e : e \in E(H)\}$ of distinct edges that is disjoint from $\cup_{v \in V(H)} E(G_v)$ such that, for every edge $e \in E(H)$ having ends u and v, the ends of f_e are contained in G_u and G_v, respectively.*



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Ask me if you want a guess:)

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Proposition

A matroid N is an induced minor of M if and only if N^* is a relaxed cc-minor of M^*.

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- **Characterize GF(q)-representable matroids that do not have $M(K_4)$ as a relaxed cc-minor.**

Relaxed cc-minor

Proposition

A matroid N is an induced minor of M if and only if N^* is a relaxed cc-minor of M^*.

- **Characterize GF(q)-representable matroids that do not have $M(K_4)$ as a relaxed cc-minor.**
- **Characterize binary matroids that does not have $M(K_4)$ or F_7 as cc-minors. In this case, we should allow simplification once we have a rank-3 matroid.**

Summary

- cc-minor is obtained by iteratively contracting cycles/circuits.
- cc-minor is the dual relation to induced restriction in general.
- cc-minor is the dual relation to induced subgraph when we enforce 2-connectivity.
- There is a way to think cc-minor in a minor perspective.
- There are some forbidden cc-minor characterizations for k-edge-connected graphs.
- A set of unavoidable cc-minors for giant 2-connected graphs are large templates.